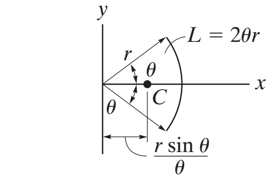
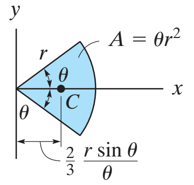
Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia



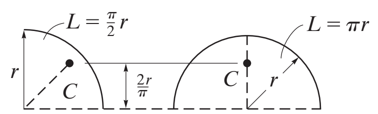
Circular arc segment



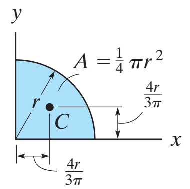
Circular sector area

$$I_x = \frac{1}{4} r^4 \left(\theta - \frac{1}{2}\sin 2\theta\right)$$

$$I_y = \frac{1}{4} r^4 \left(\theta + \frac{1}{2}\sin 2\theta\right)$$



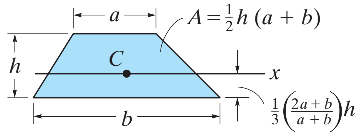
Quarter and semicircle arcs



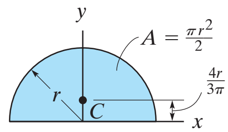
Quarter circle area

$$I_x = \frac{1}{16}\pi r^4$$

$$I_y = \frac{1}{16}\pi r^4$$



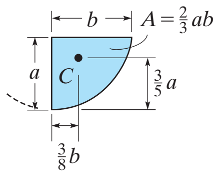
Trapezoidal area



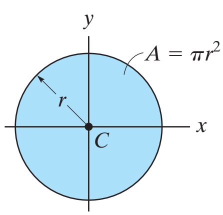
Semicircular area

$$I_x = \frac{1}{8}\pi r^4$$

$$I_y = \frac{1}{8}\pi r^4$$



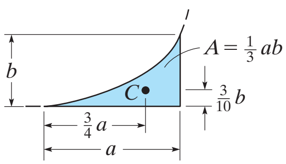
Semiparabolic area



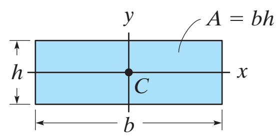
Circular area

$$I_x = \frac{1}{4}\pi r^4$$

$$I_y = \frac{1}{4}\pi r^4$$



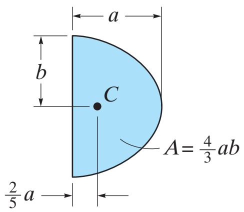
Exparabolic area



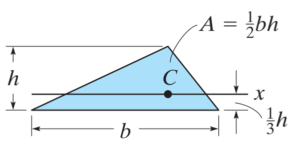
Rectangular area

$$I_x = \frac{1}{12}bh^3$$

$$I_y = \frac{1}{12}hb^3$$



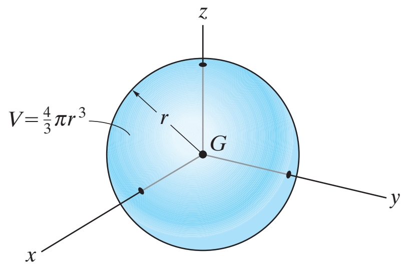
Parabolic area



Triangular area

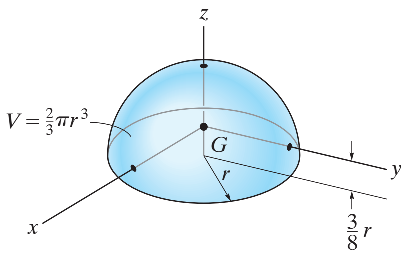
$$I_x = \frac{1}{36}bh^3$$

Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



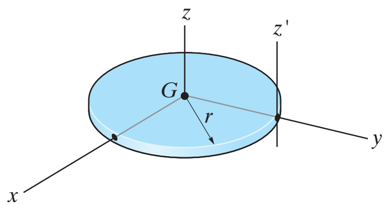
Sphere

$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$$



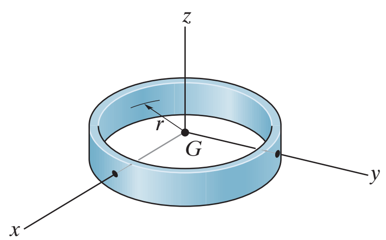
Hemisphere

$$I_{xx} = I_{yy} = 0.259\, mr^2 \qquad I_{zz} = \frac{2}{5} mr^2$$



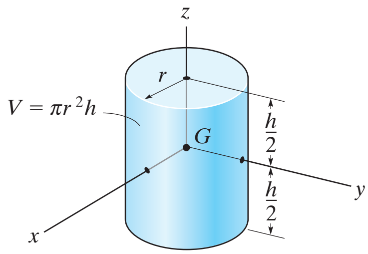
Thin Circular disk

$$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \qquad I_{zz} = \frac{1}{2} mr^2 \qquad I_{z'z'} = \frac{3}{2} mr^2$$



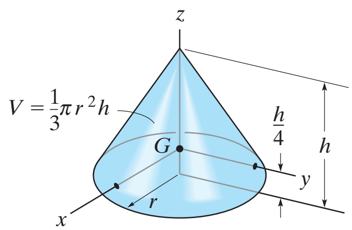
Thin ring

$$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \qquad I_{zz} = mr^2$$



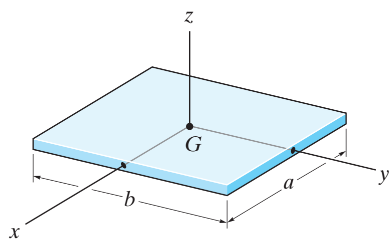
Cylinder

$$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \qquad I_{zz} = \frac{1}{2} mr^2$$



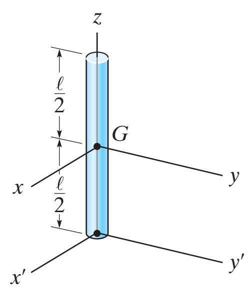
Cone

$$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \qquad I_{zz} = \frac{3}{10} mr^2$$



Thin plate

$$I_{xx} = \frac{1}{12} mb^2 \qquad I_{yy} = \frac{1}{12} ma^2 \qquad I_{zz} = \frac{1}{12} m(a^2 + b^2)$$



Slender Rod

$$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \qquad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \qquad I_{z'z'} = 0$$

Fundamental Equations of Statics

Cartesian Vector

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Directions

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

$$= \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Dot Product

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$

$$= A_xB_x + A_yB_y + A_zB_z$$

Cross Product

$$\mathbf{C} = \mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cartesian Position Vector

$$\mathbf{r} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$

Cartesian Force Vector

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

Moment of a Force

$$M_O = Fd$$

$$\mathbf{M}_O = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a Force About a Specified Axis

$$M_a = \mathbf{u}\cdot\mathbf{r}\times\mathbf{F} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Simplification of a Force and Couple System

$$\mathbf{F}_R = \Sigma\mathbf{F}$$

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

Equilibrium

Particle

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$

Rigid Body-Two Dimensions

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M_O = 0$$

Rigid Body-Three Dimensions

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$

$$\Sigma M_{x'} = 0,\ \Sigma M_{y'} = 0,\ \Sigma M_{z'} = 0$$

Friction

Static (maximum) $F_s = \mu_s N$

Kinetic $F_k = \mu_k N$

Center of Gravity

Particles or Discrete Parts

$$\bar{r} = \frac{\Sigma\,\tilde{r}\,W}{\Sigma W}$$

Body

$$\bar{r} = \frac{\int \tilde{r}\,dW}{\int dW}$$

Area and Mass Moments of Inertia

$$I = \int r^2 dA \qquad I = \int r^2 dm$$

Parallel-Axis Theorem

$$I = \bar{I} + Ad^2 \qquad I = \bar{I} + md^2$$

Radius of Gyration

$$k = \sqrt{\frac{I}{A}} \qquad k = \sqrt{\frac{I}{m}}$$

Virtual Work

$$\delta U = 0$$

SI Prefixes

Multiple	*Exponential Form*	*Prefix*	*SI Symbol*
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

Conversion Factors (FPS) to (SI)

Quantity	*Unit of Measurement (FPS)*	*Equals*	*Unit of Measurement (SI)*
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

Conversion Factors (FPS)

1 ft = 12 in. (inches)
1 mi. (mile) = 5280 ft
1 kip (kilopound) = 1000 lb
1 ton = 2000 lb